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DETERMINATION OF FLOW RATES OF ALLIS-CHALMERS AXIAL FLOW COMPRESSOR VA-312 OF PROPULSION LABORATORIES BY MEANS OF SQUARE-EDGED ORIFICES

by

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Duplicate

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1. <u>SUMMARY</u>:

This Technical Note establishes the equations necessary to calculate the flow rates through the square-edged orifice installed in the discharge pipe of the Allis-Chalmers compressor of the Propulsion Laboratory. Relations are given for the 9.0 in. and the 7.9 in. orifice diameters and the different pressure taps, namely, the flange and the Vena Contracta taps. A method is indicated to determine the tolerances of the flow measurements. It is shown also that the orifice installation satisfies ASME standards.

2. SYMBOLS:

A	Dimensionless Factor (Eq. 86, p. 212, Ref. 1)
C	Dimensional Factor (Eq. 12)
D_1	Pipe I.D. (in.)
D_2	Orifice Diameter (in.)
K	Dimensionless Discharge Coefficient at Re
K_∞	Dimensionless Discharge Coefficient at Re = ∞
P_1	Absolute Pressure ahead of Orifice (in. Hg)
P_1'	Gage Pressure ahead of Orifice (in. Hg)
R	Gas Constant (ft/$^{\circ}$R)
Re	Reynolds Number
T_1	Absolute Temperature ahead of Orifice ($^{\circ}$R)
W	Flow Rate (lb/hr)
X =	Re (10^{-6})
Y_1	Dimensionless Expansion Factor
b	Barometer (in. Hg)
h_w	Pressure Difference across Orifice (in. water)
p_1	Absolute Pressure ahead of Orifice (psia)
t_1	Temperature ahead of Orifice ($^{\circ}$F)
w	Flow Rate (lb/sec)
z'	Absolute Viscosity of Fluid (centipoise)
z =	100 z'
α	Dimensionless area multiplier
β =	D_2/D_1 Orifice Area Ratio
ζ	Dimensionless Reynolds Number Correction Factor
$\Delta(\ )$	Tolerance of Quantity in Bracket (%)

3. INSTALLATION:

Fig. 1 shows the installation of the flow orifice in the discharge pipe of the VA-312 Allis-Chalmers, 12-stage axial-flow compressor. Earlier studies showed that two orifice plates are necessary to measure the flow rate over the whole operating range of the compressor, which, because of its hydraulic speed change coupling, can run at speeds between about 4800 rpm and 12,000 rpm. The orifice plates have holes of 7.900 and 9.00 in. diameter. They meet ASME specifications and are installed in butt-welded flanges that have flange taps in accordance with ASME standards. To check the results obtained with the flange taps with those obtained with "Vena Contracta" taps, the 12 in. pipe is equipped with the pressure taps B of Fig. 1. From Ref. 1, p. 115, the upstream vena contracta tap must have a distance of one internal pipe diameter from the upstream face of the orifice plate. The location of the downstream tap depends on the orifice diameter ratio $\beta = D_2/D_1$. For D_2 = 9.00 in., β = 9.0/12.09 = 0.7445, a ratio x/D_1 = 0.4 is recommended (see Ref. 1, p. 116) where x is the distance of the tap from the downstream face of the orifice plate. The ratio x/D_1 can, however, vary between 0.28 and 0.5. Hence for D_2 = 9.0 in. the distance x was chosen as x = 5 in., or x/D_1 = 0.413. For D_2 = 7.9 in. (β = 7.9/12.09 = 0.6534) the recommended ratio is x/D_1 = 0.5, with a minimum of 0.37, and a maximum of 0.62. For the smaller orifice the distance x was therefore taken as 6 in., or x/D_1 = 0.497. The taps conform to ASME standards (see p. 117, Ref. 1). The two flange taps in either flange, and the corresponding vena contracta taps, will be connected permanently by means of 1/4" copper tubes. A valve will be arranged in each common line (see Fig. 1). Plastic hoses will connect these valves with the measuring instruments in the control room. The pressure difference h_w will be measured with either the 30 in., or the 40 in. Meriam Vernier Manometers which are filled with water. The measuring accuracy of these instruments is abt. $\pm$ 0.002 in.. The gage pressure P_1' ahead of the orifice will be measured either with the mercury filled manometer board, or a precision pressure gage. The measuring accuracy with the manometer boards is abt. $\pm$ 0.03 in., Hg., that with the gages abt. $\pm$ 0.2 in. Hg. The temperature t_1 ahead of the orifice will be measured with an instrument

that is arranged in the vertical part of the pipe after the water separator. The accuracy of the temperature measurement will be abt. $\pm$ 1°F.

Experience shows that a length $\gamma(D_1)$ of straight pipe must occur between the upstream bend of Fig. 1 and the orifice, and that a length $\delta(D_1)$ of straight pipe must be arranged after the orifice. From Ref. 1, p. 106:

for $D_2 = 9.0$ ($\beta = 0.7445$) $\gamma = 16.5$

$\delta = 4.3$

for $D_2 = 7.9$ ($\beta = 0.6534$) $\gamma = 11.2$

$\delta = 4.0$

From Fig. 1, there are:

$\gamma = 18$

$\delta = 4.3$

Hence, the straight pipe lengths of the installation of the orifice exceed the minimum requirements, and it is not necessary to install flow straighteners.

4. <u>FORMULAS</u>:

From Ref. 1, p.6, Eq. 12, the flow rate W in lb/hr is obtained from

$$W = 359.1\ (D_2^{\ 2})\ \alpha\ K\ Y_1 \sqrt{\rho_1\ h_w} \tag{1}$$

In addition to the symbols of paragraph 1, this formula contains the specific weight ρ_1 (lb/ft^3) of the gas. There is

$$\rho_1 = \frac{p_1\ 144}{R\ T_1} \tag{2}$$

where R = 53.345 ft/$^{\circ}$R for air, and p_1 in psia. Expressing p_1 by P_1, or inches of mercury,

$$p_1 = P_1\ (0.491) \tag{3}$$

For stainless steel the area multiplier α because of the thermal expansion of the orifice can be expressed by (see Ref. 1, p. 257)

$$\alpha = 1 + 0.0015 \left(\frac{t_1 - 60}{100} \right) \tag{4}$$

since t_1 will not exceed 350^oF.

The discharge coefficient K depends on β, the kind of taps used for the measuring of the pressures, and the Reynolds number Re. From Ref. 1, p. 64,

$$Re = \frac{6.316\ W}{D_2\ \ z'} \tag{5}$$

where z' is the absolute viscosity of the fluid in centipoise. The viscosity z' is independent of pressure for the present application (see Ref. 1, p. 333). Taking a linear relationship between z' and t_1 for the range of t_1 from abt. $t_1 = 100^oF$ to $t_1 = 300^oF$, there is from Ref. 1, p.335,

$$z = 100\ z' = 1.9 + 0.24 \left(\frac{t_1}{100} - 1 \right) \tag{6}$$

With $W = 3600\ w$, $D_2 = \beta\ D_1 = \beta\ (12.09)$, and $X = Re\ (10^{-6})$,

$$X = Re\ (10^{-6}) = \frac{(0.188)}{\beta}\ \frac{w}{z} \tag{7}$$

Ref. 1, p. 212, shows that the discharge coefficient K at an arbitrary Reynolds number Re can be related to the discharge coefficient K* at a Reynolds number Re* by

$$K = K^* \left(\frac{1 + A/Re}{1 + A/Re^*} \right) \tag{8}$$

where the factor A is given by Eq. 86, p.212, of Ref. 1. In particular, for $Re^* = \infty$, and $K^* = K_\infty$, let

$$K = \zeta\ K_\infty \tag{9}$$

where, with X from Eq. 7,

$$\zeta = 1 + \frac{A}{Re} = 1 + \frac{A}{X(10^6)} \tag{10}$$

Hence K at an arbitrary Reynolds number can be determined from Eqs. 9 and 10 if K_∞ at $Re = \infty$ is known. For flange taps the values of K_∞ for a 12 in. pipe are given on p.226, Ref. 1. For vena contracta taps the K values are given for finite Reynolds number on pp. 233/234 of Ref. 1 for 10 in. and 14 in. pipes. Interpolation of these data gives K* at, say, $Re^* = 10^6$. With the appropriate value of A the coefficient K_∞ is then obtained from Eq. 8.

The following results were obtained:

		Flange Taps	Vena Contracta Taps
D_2 = 9 in.	K_∞	0.7143	0.7280
	A	4660	2870
D_2 = 7.9 in.	K_∞	0.6666	0.6715
	A	3040	2250

The expansion factor Y_1 takes account of the compressibility of the fluid passing through the orifice. It is given by Eq. 60, p. 71, Ref. 1. For air, with $c_p/c_v = 1.4$, the differential pressure h_w in inches of water, and the absolute static pressure P_1 ahead of the orifice in inches of mercury, there is for flange and vena contracta taps

$$Y_1 = 1 - (0.41 + 0.35\ \beta^4)\ (9.7)\ \frac{h_w}{P_1} \qquad (11)$$

5. SUMMARY OF RELATIONS:

Combining the above-listed equations gives the following expression for the calculation of the flow rate:

$$w = C \, \alpha \, Y_1 \, \zeta \sqrt{\frac{P_1 \, h_w}{T_1}} \qquad \text{(lb/sec)} \qquad (12)$$

where C depends on the orifice diameter and the type of the pressure taps used.

The symbols in Eq. 12 represent the following quantities:

w = flow rate through orifice (lb/sec)

α = dimensionless area multiplier = $1 + 0.0015 \dfrac{(t_1 - 60)}{100}$

Y_1 = dimensionless expansion factor (Eq. 11)

ζ = dimensionless Reynolds number correction factor

P_1 = absolute pressure ahead of orifice (in.Hg.) = $P_1' + b$

P_1' = gage pressure ahead of orifice (in.Hg.) = (psig) (2.0367)

b = barometer (in.Hg.)

h_w = pressure difference across orifice (in. H_2O) = (in.Hg.) (13.59)

T_1 = absolute temperature ahead of orifice ($^{\circ}R$) = $t_1 + 460^{\circ}$

t_1 = temperature ahead of orifice ($^{\circ}F$)

The data for the 9 in. orifice are given in Table I.

Table I. (Orifice D_2 = 9.0 in.)

$Y_1 = 1 - 0.027 \dfrac{h_w}{P_1}$ $X = 0.2525 \dfrac{w}{z}$, (z from Eq. 6)	
Flange Taps	Vena Contracta Taps (12 in. ahead, 5 in. after orifice)
$C = 6.6445$ $\zeta = 1 + \dfrac{0.0047}{X}$	$C = 6.7720$ $\zeta = 1 + \dfrac{0.0029}{X}$

The necessary data for the 7.9 in. orifice are given in Table II.

Table II. (Orifice D_2 = 7.9 in.)

$Y_1 = 1-0.025 \frac{h_w}{P_1}$ $X = 0.288 \frac{w}{z}$, (z from Eq. 6)	
Flange Taps	Vena Contracta Taps (12 in. ahead, 6 in. after orifice)
$C = 4.7778$ $\zeta = 1 + \frac{0.0030}{X}$	$C = 4.8129$ $\zeta = 1 + \frac{0.0023}{X}$

Since ζ does not differ greatly from unity, the flow rate $w = w^*$ is first calculated with Eq. 12 for $\zeta = 1$. The value of X is then determined for $w = w^*$ to obtain ζ, and the actual flow rate w is

$$w = \zeta w^*$$

6. TOLERANCE OF FLOW MEASUREMENT:

If $\Delta(\ )$ symbolizes the tolerance of the quantity in the bracket in percent, the total tolerance $\Delta(w)$ of the measured flow rate, in percent, is (see Ref. 1, p.23)

$$\Delta(w) = \pm \left\{ \left[2\Delta(D_2)\right]^2 + \left[\Delta(\alpha)\right]^2 + \left[\Delta(C)\right]^2 + \left[\Delta(Y_1)\right]^2 + \left[\tfrac{1}{2}\Delta(h_w)\right]^2 + \left[\tfrac{1}{2}\Delta(P_1)\right]^2 + \left[\tfrac{1}{2}\Delta(T_1)^2\right] \right\}^{\frac{1}{2}} \quad (13)$$

Independent of the orifice diameter, there are:

$\Delta(\alpha)$ ..... $\pm$ 0.005

$\Delta(Y_1)$ .... 0 to $\pm$ 0.5 for $\frac{h_w}{P_1}$ = 0 to 2.7

$\pm$ 0.5 to $\pm$ 1 for $\frac{h_w}{P_1}$ > 2.7

$\Delta(h_w)$ .... $\pm \frac{0.002}{h_w}$ (100) for Meriam Vernier Micromanometer

$\Delta(P_1)$ .... $\pm \frac{0.03}{P_1' + b}$ (100) for Mercury Column

$\pm \frac{0.2}{(P_1' + b)}$ (100) for Precision Gage

$\Delta(T_1)$ .... $\pm \frac{100}{T_1} = \pm \frac{100}{t_1 + 460}$

Further for the 9 in. orifice:

$\Delta(D_2)$ .... $\pm \frac{0.001}{9}$ (100) = $\pm$ 0.11

$\Delta(C)$ ..... $\pm$ 1 for Flange Taps

$\pm$ 0.4 for Vena Contracta Taps

For the 7.9 in. orifice:

$\Delta(D_2)$ .... $\pm \frac{0.001}{7.9}$ (100) = $\pm$ 0.13

$\Delta(C)$ ..... $\pm$ 0.5 for Flange Taps

$\pm$ 0.4 for Vena Contracta Taps

7. REFERENCES:

1. Stearnes, R. F., et al. "FLOW MEASUREMENT WITH ORIFICE METERS", Van Nostrand Co., Inc., New York, 1951 (USNPS Reference Library No. 532.52, S7)

2. NAVDOCKS SPEC. NO 39189/61 for Astro/Aero Propulsion Laboratories, USNPS, SEC. 37, p. 9 and 23, (37.5.4.1 to 12, and 37.15.3.3)

VAVRA	USNPGS, DEPT. OF AERONAUTICS, PROPULSION LAB.	FIG. 1
	FLOW ORIFICE IN 12 IN. DISCHARGE PIPE OF	TECH. NOTE
AUGUST 63	ALLIS - CHALMERS AXIAL-FLOW COMPRESSOR	NO. 63T-2